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ON THE PRESERVATION OF COMBINATORIAL TYPES FOR MAPS ON TREES

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1. Introduction and statement of the main result.

We deal with a classical problem in combinatorial dynamics: Given a topological space X, a continuous map $f: X \to X$ and a periodic orbit $A \subset X$ of f, what can be said about the dynamics (periodic orbits, topological entropy—see [1]—, etc.) of f in terms of $f_{|A}$?

A well known case is when X is a closed interval $I \subset \mathbb{R}$. Indeed, if $f: I \to I$ is a continuous map then intrinsic information can be obtained by considering the *"pattern"* of A which is characterized by the permutation π_A induced by $f_{|A}$ (see [7] or [9]). To each pattern π we may associate a map f_π which has a finite invariant set B such that the permutation induced by $f_{\pi|B}$ is π and f_π is monotone between consecutive points of B (a "connect-the-dots" map). Such a map is called a π-*monotone model* and its existence has some important consequences. During the 1980's and the early 1990's, monotone models for interval maps (and a trivial generalization to star maps) were used by several authors to tackle a wide variety of problems (see for instance [4], [5], [7], [8], [9] and [10]).

We are interested in continuous maps defined on trees (from now on, such a map will be called a *tree map*). In [2], the authors introduce a notion of *pattern* of a finite invariant set of a tree map, and prove that for any

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