TRANSACTIONS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 351, Number 4, April 1999, Pages 1551–1573
S 0002-9947(99)02077-2

ENTROPY AND PERIODIC POINTS FOR TRANSITIVE MAPS

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Dedicated to the memory of Wiesław Szlenk

ABSTRACT. The aim of this paper is to investigate the connection between transitivity, density of the set of periodic points and topological entropy for low dimensional continuous maps. The paper deals with this problem in the case of the n–star and the circle among the one–dimensional spaces and in some higher dimensional spaces. Particular attention is paid to triangular maps and to extensions of transitive maps to higher dimensions without increasing topological entropy.

1. INTRODUCTION AND MAIN RESULTS

Recently, several authors became interested in studying the dynamics of maps of the interval, the circle, the Y and the n–star (see for instance [6], [15], [5], [11]) and the dynamics of tree maps (see for instance [12], [34], [8]). The interest in studying maps on such one–dimensional spaces is due to the fact that for maps on manifolds with an invariant foliation of codimension one, the corresponding quotient map turns out to be defined in general on a graph. Furthermore, the dynamics of pseudo-Anosov homeomorphisms on a surface can be essentially reduced to the analysis of some special graph maps (see e.g. [27]). Finally, a graph map sometimes imitates the behavior of a smooth map (flow) in a neighborhood of a hyperbolic attractor (see for instance [43]). One of the main reasons to study the transitive one–dimensional maps is the fact that the invariant subsets of transitive maps are models for the ω-limit sets of arbitrary maps on which the entropy is positive (see [20]).

Triangular maps are close to one–dimensional maps in the sense that some important dynamical features extend to triangular maps. On the other hand, they already display other important properties which are typical for higher dimensional maps and cannot be found in the one–dimensional maps (see for instance [29], [4], [30], [2]). We will show that this dualism can also be found in the study of the transitive triangular maps.

Let $f : X \longrightarrow X$ be a continuous map of a compact metric space. We say that f is *(topologically) transitive* if for any two non-empty open sets U and V in X, there is a nonnegative integer k such that $f^k(U) \cap V \neq \emptyset$. If X has no isolated points, then the above definition is equivalent to the following one. The map f is transitive if it has a dense orbit, i.e. if there exists $x \in X$ such that the orbit of x,

Received by the editors September 20, 1995 and, in revised form, November 26, 1996.
1991 *Mathematics Subject Classification.* Primary 58F11, 34C35, 54H20.
Key words and phrases. Transitive maps, topological entropy, periodic points, triangular maps.