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Axiomatic definition of the topological entropy on the interval

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Summary. The aim of this paper is to give an axiomatic definition of the topological entropy for continuous interval maps and, in such a way, to shed some more light on the importance of the different properties of the topological entropy in this setting. We give two closely related axiomatic definitions of topological entropy and an axiomatic characterization of the topological chaos.

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1. Introduction

The *topological entropy* as a measure of the complexity of a continuous self-map of a compact topological space was introduced by Adler, Konheim and McAndrew [1] and has been studied by many authors. In particular, in compact metric spaces an equivalent definition has been found by Bowen [8] and Dinaburg [10]. For the definition and main properties of topological entropy we refer the reader to [2] and [21] (the book [2] is particularly focused on interval maps and very often, when we use well known facts, we refer the reader to this book instead to the original paper).

The idea of giving an *axiomatic* definition of entropy belongs to Rohlin who gave in [16] an axiomatic definition of the measure-theoretic entropy of an automorphism of a Lebesgue space. Later an analogue of Rohlin's result for a $\mathbb{Z}^d$-action for every $d \geq 2$ was proved by Kamiński in [13]. An axiomatic definition of topological entropy for endomorphisms on compact groups was found by Stojanov [20]. For completeness we recall that there are also many papers dealing with axiomatic characterizations of various kinds of entropy in the context of information theory.

Our main concern in this paper is to give an axiomatic definition of the topological entropy for continuous interval maps and, in such a way, to shed some more light on the importance of the different properties of the topological entropy in this