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ON TOPOLOGICAL ENTROPY OF TRIANGULAR MAPS OF THE SQUARE

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We study the topological entropy of triangular maps of the square. We show that such maps differ from the continuous maps of the interval because there exist triangular maps of the square of "type 2^∞" with infinite topological entropy. The set of such maps is dense in the space of triangular maps of "type at most 2^∞" and the topological entropy as a function of the triangular maps of the square is not lower semicontinuous. However, we show that for these maps the characterisation of the lower bounds of the topological entropy depending on the set of periods is the same as for the continuous maps of the interval.

1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

For a compact topological space X, denote by $C(X,X)$ the set of all continuous self-maps of X. The dynamical system given by a map $G \in C(X,X)$ is called *topologically chaotic* or *simple* if the topological entropy $h(G)$ of G is positive or zero, respectively. A natural question is: which properties characterise the topologically chaotic dynamical systems?

If I is a compact interval of the real line and $f \in C(I,I)$, then the topological entropy of f is zero if and only if the period of any periodic point of f is a power of two (see [**7, 21**]). To state this result in other words, denote by $\mathbb{N}$ the set of all positive integers and introduce the *Sharkovskiĭ ordering* ${}_{s}\!>$ on the set $\mathbb{N} \cup \{2^\infty\}$ by:

$$3 \,{}_{s}\!> 5 \,{}_{s}\!> 7 \,{}_{s}\!> \dots \,{}_{s}\!> 2\cdot 3 \,{}_{s}\!> 2\cdot 5 \,{}_{s}\!> 2\cdot 7 \,{}_{s}\!> \dots \,{}_{s}\!> 4\cdot 3 \,{}_{s}\!> 4\cdot 5 \,{}_{s}\!> 4\cdot 7 \,{}_{s}\!> \dots \,{}_{s}\!> \dots$$
$$\,{}_{s}\!> 2^n\cdot 3 \,{}_{s}\!> 2^n\cdot 5 \,{}_{s}\!> 2^n\cdot 7 \,{}_{s}\!> \dots \,{}_{s}\!> 2^\infty \,{}_{s}\!> \dots \,{}_{s}\!> 2^n \,{}_{s}\!> \dots \,{}_{s}\!> 16 \,{}_{s}\!> 8 \,{}_{s}\!> 4 \,{}_{s}\!> 2 \,{}_{s}\!> 1.$$

We shall also use the symbol ${}_{s}\!\geqslant$ in the natural way. For $s \in \mathbb{N} \cup \{2^\infty\}$ we denote by $S(s)$ the set $\{k \in \mathbb{N} : s \,{}_{s}\!\geqslant k\}$ ($S(2^\infty)$ stands for the set $\{1, 2, 4, \dots, 2^k, \dots\}$) and by

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