



## LOW-DIMENSIONAL COMBINATORIAL DYNAMICS

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The aim of this paper is to give an account of some of the progress made in these last years in the combinatorial low-dimensional dynamics and to suggest some research directions and open problems.

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### 1. Introduction

The theory of low-dimensional combinatorial dynamics started with the publication of the Sharkovskii’s Theorem (see Sec. 2.1). It describes the possible sets of periods of all periodic orbits of a continuous self-map of the interval. The whole theory which was developed starting with these initial results, deals mainly with combinatorial objects,

permutations, graphs, etc. In [Alsedà *et al.*, 1993] we decided to call it *combinatorial dynamics*. An important part in this theory is also played by *topological entropy*. It is an important measure of the complexity of a dynamical system, or of the degree of “chaos” present in it.

The objective of this tutorial paper is to give a partial account of the progress made in some directions during these last years in the combinatorial

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