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KNEADING THEORY FOR A FAMILY OF CIRCLE MAPS WITH ONE DISCONTINUITY

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Abstract. We apply the kneading theory techniques to a class of circle maps with one discontinuity and we characterize the rotation interval of a map in terms of the kneading sequences. As a consequence we obtain lower and upper bounds of the entropy depending on the rotation interval.

1. Introduction

We study the class $\mathcal{C}$ of maps $F\colon \mathbf{R} \longrightarrow \mathbf{R}$ defined as follows (see Figure 1). We say that $F \in \mathcal{C}$ if:

(1) $F|_{(0,1)}$ is bounded, continuous and non-decreasing.
(2) $\lim_{x\uparrow 1} F(x) > \lim_{x\downarrow 1} F(x)$.
(3) $F(x+1) = F(x) + 1$ for all $x \in \mathbf{R}$.

For a map $F \in \mathcal{C}$ and for each $a \in \mathbf{Z}$ we set $F(a^+) = \lim_{x\downarrow a} F(x)$ and $F(a^-) = \lim_{x\uparrow a} F(x)$. In view of (3) we have $F(a^+) = F(0^+) + a$ and $F(a^-) = F(0^-) + a$. Note that the exact value of $F(0)$ is not specified. Then in what follows we consider that $F(0)$ is either $F(0^+)$ or $F(0^-)$, or both, as necessary.

Since every map $F \in \mathcal{C}$ has a discontinuity in each integer, the class $\mathcal{C}$ can be considered as a family of liftings of circle maps with one discontinuity.

The maps of class $\mathcal{C}$ appear in a natural way in the study of many branches of dynamics. The simplest example of such maps is the family $x \to \beta x + \alpha$, which plays an important role in ergodic theory (see [**H**]). The case $\alpha = 0$ gives the famous β-transformations (see [**R**]). Also, the class $\mathcal{C}$ contains the class of the Lorenz-Like maps which has been studied by several authors (see [**ALMT**], [**G**], [**GS**], [**Gu**], [**HS**], [**S**]).

The aim of this paper is to extend the kneading theory developed in [**AM**] for continuous maps of the circle of degree one to class $\mathcal{C}$, to obtain a characterization of the rotation interval of a map in terms of its kneading sequences. From this characterization we shall obtain models with maximum and minimum entropy

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