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Forcing and entropy of strip patterns of quasiperiodic skew products in the cylinder ☆

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ABSTRACT

We extend the results and techniques from [7] to study the combinatorial dynamics (*forcing*) and entropy of quasiperiodically forced skew-products on the cylinder. For these maps we prove that a cyclic permutation τ forces a cyclic permutation ν as interval patterns if and only if τ forces ν as cylinder patterns. This result gives as a corollary the Sharkovskiĭ Theorem for quasiperiodically forced skew-products on the cylinder proved in [7]. Next, the notion of s-horseshoe is defined for quasiperiodically forced skew-products on the cylinder and it is proved, as in the interval case, that if a quasiperiodically forced skew-product on the cylinder has an s-horseshoe then its topological entropy is larger than or equals to $\log(s)$. Finally, if a quasiperiodically forced skew-product on the cylinder has a periodic orbit with pattern τ, then $h(F) \geq h(f_\tau)$, where f_τ denotes the *connect-the-dots* interval map over a periodic orbit with pattern τ. This implies that if the period of τ is $2^n q$ with $n \geq 0$ and $q \geq 1$ odd, then $h(F) \geq \frac{\log(\lambda_q)}{2^n}$, where $\lambda_1 = 1$ and, for each $q \geq 3$, λ_q is the largest root of the polynomial $x^q - 2x^{q-2} - 1$. Moreover, for every $m = 2^n q$ with $n \geq 0$ and $q \geq 1$ odd, there exists a quasiperiodically forced skew-product on the cylinder F_m with a periodic orbit of period m such that $h(F_m) = \frac{\log(\lambda_q)}{2^n}$. This extends the analogous result for interval maps to quasiperiodically forced skew-products on the cylinder.

1. Introduction

In this paper we want to study the coexistence and implications between periodic objects of maps on the cylinder $\Omega = \mathbb{S}^1 \times \mathbb{I}$, of the form:

$$F : \begin{pmatrix} \theta \\ x \end{pmatrix} \longrightarrow \begin{pmatrix} R_\omega(\theta) \\ f(\theta, x) \end{pmatrix},$$

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