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# A quasiperiodically forced skew-product on the cylinder without fixed-curves

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ABSTRACT

In Fabbri et al. (2005) the Sharkovskiĭ Theorem was extended to periodic orbits of strips of quasiperiodic skew products in the cylinder.

In this paper we deal with the following natural question that arises in this setting: *Does Sharkovskiĭ Theorem hold when restricted to curves instead of general strips?*

We answer this question in the negative by constructing a counterexample: We construct a map having a periodic orbit of period 2 of curves (which is, in fact, the upper and lower circles of the cylinder) and without any invariant curve.

In particular this shows that there exist quasiperiodic skew products in the cylinder without invariant curves.



## 1. Introduction

We consider the coexistence and implications between periodic objects of maps on the cylinder $\Omega = \mathbb{S}^1 \times \mathbb{I}$, of the form:

$$T\colon \begin{pmatrix} \theta \\ x \end{pmatrix} \longrightarrow \begin{pmatrix} R_\omega(\theta) \\ \zeta(\theta, x) \end{pmatrix},$$

where $\mathbb{S}^1 = \mathbb{R}/\mathbb{Z}$, $\mathbb{I}$ is a compact interval of the real line, $R_\omega(\theta) = \theta + \omega \pmod 1$ with $\omega \in \mathbb{R}\backslash\mathbb{Q}$ and $\zeta(\theta, x) = \zeta_\theta(x)$ is continuous on both variables. The class of all maps of the above type will be denoted by $\mathcal{S}(\Omega)$.

In this setting a very basic and natural question is the following: *is it true that any map in the class $\mathcal{S}(\Omega)$ has an invariant curve?*

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