CONTINUITY OF ENTROPY FOR BIMODAL MAPS

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ABSTRACT

We characterize the continuity of the topological entropy of bimodal maps of the interval and of the circle in terms of the behaviour of the iterates of the turning points and of the value of the topological entropy of the map under consideration. In the case of bimodal circle maps of degree one we also study the continuity of the entropy in terms of their rotation intervals

1. *Introduction*

Let $\mathscr{I}$ and $\mathscr{S}$ denote the spaces of all continuous maps from a closed proper interval of the real line into itself and of continuous maps of the circle into itself, respectively. We shall consider these spaces to be endowed with the topology of the uniform convergence. We recall that the topological entropy in these spaces is lower semicontinuous. In [**11**] Misiurewicz computed the maximum level up to which the topological entropy can jump at a given continuous map in the class of maps of $\mathscr{I}$ or $\mathscr{S}$ with a fixed number of turning points. Then if the topological entropy of a map f of $\mathscr{I}$ or $\mathscr{S}$ is larger than or equal to the maximum jump allowed at this map, we deduce that the entropy is continuous at f. Furthermore, he also obtained as a corollary that the entropy is continuous whenever it is positive in the subspace of $\mathscr{I}$ consisting of all unimodal maps.

The aim of this paper is to use Misiurewicz's result to characterize the continuity of the topological entropy of bimodal maps of $\mathscr{I}$ or $\mathscr{S}$ in terms of the behaviour of the iterates of the turning points and the value of the topological entropy of the map under consideration. In the case of bimodal circle maps of degree one we shall also discuss the continuity of the entropy in terms of their rotation intervals.

The paper is organized as follows. In the next section we fix the notation and state some preliminary results for interval maps. Then in Section 3 we study the continuity of the entropy for bimodal interval maps. Sections 4 and 6 are devoted to introducing the necessary definitions and preliminary results on circle maps, while in Sections 5 and 7 we study the continuity of bimodal circle maps of degree different from one and of degree one, respectively. Finally in Section 8 we study the continuity of the entropy of bimodal circle maps of degree one in terms of their rotation intervals.

2. *Definitions and preliminary results for interval maps*

This section will be devoted mainly to stating Misiurewicz's Theorem [**11**] on the jumps of entropy of interval maps. To this end we introduce some notation.

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