Ergod. Th. & Dynam. Sys. (2000), **20**, 1559–1576
Printed in the United Kingdom

Minimizing topological entropy for continuous maps on graphs

LL. ALSEDÀ†, F. MAÑOSAS† and P. MUMBRÚ‡

†*Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08913 Cerdanyola del Vallès, Barcelona, Spain*
(e-mail: alseda@mat.uab.es, manyosas@mat.uab.es)
‡*Departament de Matemàtica Aplicada i Anàlisi, Universitat de Barcelona, Gran Via 585, 08071 Barcelona, Spain*

(*Received* 12 *June* 1998 *and accepted in revised form* 22 *March* 1999)

Abstract. We study the existence of models with minimal topological entropy among the class of all continuous maps from a given graph to itself with a fixed behavior on a given finite invariant set.

1. *Introduction*

A classical problem in dynamics is the following one. Assume that X is a topological space and that f is a map from X to itself. Assume also that we know f restricted to an invariant set $A \subset X$. What can be said about the dynamics of f? Moreover, does there exist a map exhibiting the same behavior as $f|_A$ and having minimal dynamics in some sense (i.e. minimal topological entropy, minimal set of periods or periodic orbits, minimal set of 'types' of orbits, ... ?).

When the space X is a closed interval of the real line and A is a finite set the answer to the above problem is well known (see [**10**]). Indeed, the 'connect-the-dots' map is the one exhibiting minimal dynamics in all senses mentioned above. A generalization of this problem when A is not invariant can be found in [**3**].

In [**4**] this problem is considered when the space is a tree and A is a finite invariant set. In that paper it was shown that there is an important difference between the interval and the tree case (see for instance [**4**, Example 1.1]). Namely, to obtain models with minimal dynamics it is necessary to modify the tree (here minimal dynamics means having a minimal set of 'types' of orbits and, consequently, minimal topological entropy).

From above, a natural question is the following. If we do not allow the space to be modified is there another criterion of minimal dynamics for which there exist minimal models?

The aim of this paper is to study this problem when the criterion for minimal dynamics is minimal topological entropy and the space is a graph. In particular, we prove (see the Main