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Asymptotic linking of periodic orbits for diffeomorphisms of the two-disk

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Abstract. Let f be a $C^{1+\epsilon}$ orientation preserving diffeomorphism of the two-disk with positive topological entropy. We define for f an interval of topological invariants. Each point in this interval describes the way the elements of an infinite sequence of periodic orbits with arbitrarily large periods, are asymptotically linked one around the other.

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1. Introduction

Transverse homoclinic points for surface diffeomorphisms were first discovered by Poincaré who also noticed their connection with complicated dynamics: by invariance of the stable and unstable manifolds of hyperbolic periodic points, transverse homoclinic points come in infinite bunches. This connection was made more precise by Birkhoff [Bi] who proved that transverse homoclinic points imply the existence of infinitely many periodic orbits. Later, Smale [Sm] described how transverse homoclinic points imply that the action of high iterates of the map, on a small rectangle based on the stable manifold of the corresponding saddle point, can be modelled by horseshoe maps (see section 2), and Katok [Ka] proved that for a smooth diffeomorphism, the existence of transverse homoclinic points is a necessary condition for positive topological entropy (see section 3).

If we restrict ourselves to the case of orientation preserving diffeomorphisms of the two-disk, we can go further in the analysis of this infinity of periodic orbits created by transverse homoclinic points. A pioneering work in this direction has been done by Holmes and Williams [HW] who investigated the knot types of periodic orbits for a standard suspension of the horseshoe map. In this description a special class of infinite sequences of periodic orbits appear. These sequences occur naturally in two-dimensional dynamics, for instance for conservative maps or for maps at the

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