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Attractors for unimodal quasiperiodically forced maps

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Dedicated to Gerry Ladas on the occasion of his 70th Birthday

We consider unimodal quasiperiodically forced maps, that is, skew products with irrational rotations of the circle in the base and unimodal interval maps in the fibres: the map in the fibre over θ is a unimodal map f of the interval $[0, 1]$ onto itself multiplied by $g(\theta)$, where g is a continuous function from the circle to $[0, 1]$. Here we consider the 'pinched' case, when g attains the value 0. This case is similar to the one considered by Gerhard Keller, except that the function f in his case is increasing. Since in our case f is unimodal, the basic tools from the Keller's paper do not work in general.

We prove that under some additional assumptions on the system there exists a *strange nonchaotic attractor*. It is the graph of a measurable function from the circle to [0,1], which is invariant, discontinuous almost everywhere and attracts almost all trajectories. Moreover, both Lyapunov exponents on this attractor are nonpositive. There are also cases when the dynamics is completely different, because one can apply the results of Jerome Buzzi implying the existence of an invariant measure absolutely continuous with respect to the Lebesgue measure (and then the attractor is some region in $\mathbb{S}^1 \times [0, 1]$), and the maximal Lyapunov exponent is positive. Finally, there are cases when we can only guess what the behaviour is by performing computer experiments.

Keywords: quasiperiodically forced system; strange nonchaotic attractor; logistic map; Lyapunov exponents

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1. Introduction

In this paper, we deal with nonautonomous quasiperiodically forced dynamical systems given by a map $F : \mathbb{S}^1 \times X \to \mathbb{S}^1 \times X$:

$$F(\theta, x) = (R(\theta), \psi(\theta, x)), \tag{1.1}$$

where X is a topological space, $\mathbb{S}^1 = \mathbb{R}/\mathbb{Z}$ denotes the unit circle and $R : \mathbb{S}^1 \to \mathbb{S}^1$ denotes the rotation of the circle by angle ω (that is, $R(\theta) = \theta + \omega \,(\mathrm{mod}\, 1)$), where ω is an irrational number.

Such a system is called a *skew product*. Then, the map $R : \mathbb{S}^1 \to \mathbb{S}^1$ is called the *base map* and $\{\theta\} \times X$ is called the *fibre over* θ. For a fixed θ, the map

$$\psi(\theta, \cdot) : \{\theta\} \times X \to \{R(\theta)\} \times X,$$

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