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SHORT NOTE

A note on a rational difference equation

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We answer a question raised by G. Ladas at the *ICDEA 2009* conference, by showing that the non-autonomous difference equation $x_{n+1} = 1/(x_n + A_n)$ with x_n, $A_n > 0$ and $A_n \to 0$ with the ratios A_{n+1}/A_n bounded can have solutions whose set of accumulation points contain a non-degenerate interval.

Keywords: difference equation; rational; non-autonomous; solutions; non-convergence

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1. Introduction

At the *ICDEA 2009* conference, G. Ladas raised the question whether the solutions of the non-autonomous difference equation

$$x_{n+1} = \frac{1}{x_n + A_n}, \tag{1}$$

where

$$x_n, A_n > 0, \quad A_n \to 0,$$

with the ratios A_{n+1}/A_n bounded, have to converge to a period 2 or 1 cycle.

We will show that the answer to this question is negative. Namely, the following theorem holds.

THEOREM 1.1. *Let t, s be real numbers such that $1 < s < t < \sqrt{2}$. Then, there exists a sequence (A_n) of positive numbers, with $A_n \to 0$ and the ratios A_{n+1}/A_n taking only values 1/2 and 2, such that the difference equation* (1) *has a positive solution (x_n) for which the set of accumulation points of the sequence (x_{2n}) is equal to the interval $[s, t]$.*

2. Proof of the theorem

We fix s and t as above, $x_0 \in [s,t]$, and $A_0 \in (0,1/9)$ such that $t(A_0 + t) < 2$. Then, for each n, we define A_{2n+1} and A_{2n+2} by induction, setting:

(a) $A_{2n+1} = A_{2n}/2$ and $A_{2n+2} = 2A_{2n+1}$, or

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