RANDOM INTERVAL HOMEOMORPHISMS

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Abstract. We investigate homeomorphisms of a compact interval, applied randomly. We consider this system as a skew product with the two-sided Bernoulli shift in the base. If on the open interval there is a metric in which almost all maps are contractions, then (with mild additional assumptions) there exists a global pullback attractor, which is a graph of a function from the base to the fiber. It is also a forward attractor. However, the value of this function depends only on the past, so when we take the one-sided shift in the base, it disappears. We illustrate those phenomena on an example, where there are two piecewise linear homeomorphisms, one moving points to the right and the other one to the left.

1. Introduction

In this paper we investigate the properties of the systems of randomly applied orientation preserving homeomorphisms of the compact interval $[0,1]$. Such a system can be considered as a skew product with a mixed topological-measure structure. In the base we do not need any topology (although sometimes we have it), but we assume that we have there an ergodic measure preserving transformation of a probability space. In the fiber, which is an interval, we have orientation preserving homeomorphisms, depending in a measurable way on the point in the base.

We are interested in the existence of almost global attractors which are graphs of measurable functions from the base to the fiber. When we speak of an attractor, we mean a set towards which almost all orbits converge, and the convergence is considered fiberwise (only in the direction of a fiber). This agrees with the philosophy saying that the phase space is really only the fiber space (here, the interval).

Those systems and their attractors can be looked upon from various points of view (random systems, Strange Nonchaotic Attractors, Iterated Function Systems, nonautonomous systems, etc.), see [1].

Our main result is a detailed description of the behavior of a certain one-parameter family of piecewise linear random homeomorphisms. However, we precede it with some general results, which can be applied to very general random systems of interval homeomorphisms.

Note that 0 and 1 are fixed points of all orientation preserving homeomorphisms of $[0,1]$, so the products of the base space with $\{0\}$ and with $\{1\}$ (we will refer to those sets as *level* 0 and *level* 1) are invariant for the skew product. if they are attracting in the sense of negative fiberwise Lyapunov exponent, one expects their

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