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SKEW PRODUCT ATTRACTORS AND CONCAVITY

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ABSTRACT. We propose an approach to the attractors of skew products that tries to avoid unnecessary structures on the base space and rejects the assumption on the invariance of an attractor. When nonivertible maps in the base are allowed, one can encounter the mystery of the vanishing attractor. In the second part of the paper, we show that if the fiber maps are concave interval maps, then contraction in the fibers does not depend on the map in the base.

1. INTRODUCTION

We want to propose a unified approach to many situations where attractors for skew products are considered. This includes random systems, nonautonomous systems, strange nonchaotic attractors, etc. (we allow the reader to continue this list, warning that the terminology may vary). This approach is built on existing ideas, but contains two new ingredients. The first one is a realization that the space can have various structures, so one should consider which results require which structures. The second one is the introduction of the possibility that an attractor is not invariant. Moreover, admitting the possibility that the base map is nonivertible, we encounter "the mystery of the vanishing attractor". An attractor present for an invertible map in the base, vanishes when we forget about the past and replace the base map by a noninvertible one. This happens in spite of the fact that the future dynamics do not depend on the past. From the philosophical point of view this can be interpreted as an application of Mathematics to History: even if our future depends only on our present, making predictions is much more consistent when we take our past into account.[1]

In the second part of the paper we show how changing the point of view allows us to pinpoint the real reasons for existence of a Strange Nonchaotic Attractor if the fibers are one-dimensional and the maps in the fibers are concave.

We concentrate on the discrete case, that is, on iterates of self-maps of some space. Thus, we consider a skew product on a space $X = B \times Y$. The space B is the *base* and Y is the *fiber space*; for each $\vartheta \in B$ the set $\{\vartheta\} \times Y$ is the *fiber over* ϑ. We will denote by $\pi_2 : X \to Y$ the projection $\pi_2(\vartheta, x) = x$. The skew product

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[1]We hope that this remark will allow us to present this paper to the administrators in our universities as a result of the interdisciplinary research.