DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **20**, Number **10**, December **2015**

doi:10.3934/dcdsb.2015.20.3403

pp. **3403–3413**

SEMICONJUGACY TO A MAP OF A CONSTANT SLOPE

Lluís Alsedà

Departament de Matemàtiques
Edifici Cc
Universitat Autònoma de Barcelona
08913 Cerdanyola del Vallès, Barcelona, Spain

Michał Misiurewicz

Department of Mathematical Sciences
Indiana University – Purdue University Indianapolis
402 N. Blackford Street, Indianapolis, IN 46202, USA

Abstract. It is well known that a continuous piecewise monotone interval map with positive topological entropy is semiconjugate to a map of a constant slope and the same entropy, and if it is additionally transitive then this semiconjugacy is actually a conjugacy. We generalize this result to piecewise continuous piecewise monotone interval maps, and as a consequence, get it also for piecewise monotone graph maps. We show that assigning to a continuous transitive piecewise monotone map of positive entropy a map of constant slope conjugate to it defines an operator, and show that this operator is not continuous.

1. **Introduction.** It has been known for a long time that a continuous piecewise monotone interval map with positive topological entropy is semiconjugate to a map of a constant slope and the same entropy [7], and if it is additionally transitive, then it is conjugate to a map of a constant slope and the same entropy [12]. The proof by Milnor and Thurston uses kneading theory and is quite complicated. In the book [1] we gave a simpler proof, that does not use kneading theory. Here we present a modification of this proof, that works for piecewise continuous piecewise monotone interval maps, and consequently, also for piecewise monotone graph maps.

At the end we show that assigning to a continuous transitive piecewise monotone map of positive entropy a map of constant slope conjugate to it defines an operator and show that this operator is not continuous.

To avoid misunderstanding about terminology, let us mention that we will be speaking about *increasing* (resp. *decreasing, monotone*) and *strictly increasing* (resp. *strictly decreasing, strictly monotone*) maps. Also, when f is piecewise affine we define the *slope of* f as $|f'(x)|$ for every x where f is differentiable.

We will denote by $\mathcal{PM}$ the class of interval maps which are piecewise continuous piecewise monotone (with finitely many pieces), and by $\mathcal{PSM}$ the class of all maps from $\mathcal{PM}$ that are piecewise strictly monotone. The interval we consider is compact and we will denote it I. The maximal intervals where a map f is simultaneously

2010 *Mathematics Subject Classification.* Primary: 37E05, 37B40.

Key words and phrases. Piecewise monotonotone maps, semiconjugacy to a map of constant slope, topological entropy, interval Markov maps, measure of maximal entropy.

The first author has been partially supported by the MEC grant numbers MTM2008-01486 and MTM2011-26995-C02-01.