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A numerical estimate of the regularity of a family of Strange Non-Chaotic Attractors

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HIGHLIGHTS

- We give an algorithm to compute regularities of SNA's based on tools of de la Llave–Petrov.
- It uses the Keller convergence construction to the attractor.
- It uses Daubechies Wavelets with 16 vanishing moments.
- The precision is two decimal digits compared with Weierstraß function.
- The loss of regularity as parameter changes is observed from wavelet coefficients.

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ABSTRACT

We estimate numerically the regularities of a family of Strange Non-Chaotic Attractors related with one of the models studied in (Grebogi et al., 1984) (see also Keller, 1996). To estimate these regularities we use wavelet analysis in the spirit of de la Llave and Petrov (2002) together with some ad-hoc techniques that we develop to overcome the theoretical difficulties that arise in the application of the method to the particular family that we consider. These difficulties are mainly due to the facts that we do not have an explicit formula for the attractor and it is discontinuous almost everywhere for some values of the parameters. Concretely we propose an algorithm based on the Fast Wavelet Transform. Also a quality check of the wavelet coefficients and regularity estimates is done.

1. Introduction

The aim of this paper is to develop techniques and algorithms to compute approximations of (geometrically) extremely complicated dynamical invariant objects by means of wavelet expansions. Moreover, from the wavelet coefficients we want to derive an estimate of the regularity of these invariant objects. In the case when the theoretical regularity is known, the comparison between both values gives a natural and good quality test of the algorithms and approximations.

In this paper the invariant objects that we study and consider when developing our algorithms are Strange Non-chaotic Attractors. They appear in a natural way in families of quasiperiodically forced skew products on the cylinder of the form

$$\mathfrak{F}_{\sigma,\varepsilon}: \begin{array}{ccc} \mathbb{S}^1 \times \mathbb{R} & \longrightarrow & \mathbb{S}^1 \times \mathbb{R} \\ (\theta, x) & \longmapsto & (R_\omega(\theta), F_{\sigma,\varepsilon}(\theta, x)), \end{array} \tag{1}$$

where $F_{\sigma,\varepsilon}: \mathbb{S}^1 \times \mathbb{R} \longrightarrow \mathbb{R}$ is continuous and $\mathcal{C}^1$ with respect to the second variable, $R_\omega(\theta) = \theta + \omega \pmod 1$ with $\omega \in \mathbb{R} \setminus \mathbb{Q}$, $\mathbb{S}^1 = \mathbb{R}/\mathbb{Z} = [0, 1)$ denotes the circle and $\varepsilon, \sigma \in \mathbb{R}^+$. These systems have the important property that any fibre, $\{\theta\} \times \mathbb{R}$, is mapped into another fibre, $\{R_\omega(\theta)\} \times \mathbb{R}$.

Our main goal will be to derive approximations in terms of wavelets of the invariant maps $\varphi: \mathbb{S}^1 \longrightarrow \mathbb{R}$: $\varphi(R_\omega(\theta)) = F_{\sigma,\varepsilon}(\theta, \varphi(\theta))$. Under certain conditions the graphs of these invariant maps have very complicated geometry where roughly speaking, the word *complicated* means non-piecewise continuous. In such case, we will say that the graph of φ is a *Strange Non-chaotic Attractor (SNA)*. A usual particular case of SNA is when the invariant function is positive in a set of full Lebesgue measure and vanishes on a residual set.

A standard approach is to use Fourier expansions (rather than wavelet ones) when approximating dynamical invariant objects. In the SNA's framework this approach has a serious drawback: an accurate approximation of φ demands a high number of Fourier modes due to the appearance of strong oscillations (see e.g. [1]). One natural way to overcome this problem is by using other orthonormal bases such as wavelets and the multi-scale methods

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