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Linear Orderings and the Full Periodicity Kernel for the n-Star

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We show that Baldwin's characterization of the set of periods of continuous self maps of the n-star can be expressed in terms of a finite number of linear orderings. Additionally we study the minimal sets of periods which force a continuous self map of the n-star to have periodic points of all periods.

1. INTRODUCTION

Baldwin (see [3]), in an interesting paper which extends Sharkovskiĭ 's Theorem to the n-star, has shown that the set of periods of a continuous map of the n-star into itself can be expressed as the union of "initial segments" of a finite set of partial orderings of the natural numbers. On the other hand, in [1] it was shown that for the class of continuous maps of the 3-star into itself which leave the branching point fixed, the set of periods can be expressed as "initial segments" of three *linear* orderings (one of which was Sharkovskiĭ's ordering). In [2] it was noted that these three orderings can be thought of as certain orderings associated to the rationals 1/2 and 1/3. The aim of this paper is to show that this is in fact the general situation. Namely, we show that the set of periods of a continuous self map of the n-star can be expressed as the union of "initial segments" of the linear orderings associated to all rationals in the interval $(0, 1)$ with denominator smaller than or equal to n defined in certain subsets of the natural numbers. This gives a constructive proof of Theorem 1.6 of [3] which, in particular, proves Conjecture 13.4 of [1].

The fact that it is possible to characterize the sets of periods of continuous self maps of the n-star in terms of linear orderings associated to rationals, suggests that the sets of periods of such maps may arise in some way from "rotation intervals"; see [2] where an example of such

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