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COFINITENESS OF THE SET OF PERIODS FOR TOTALLY TRANSITIVE TREE MAPS

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The aim of this paper is to show that the cofiniteness of the set of periods characterizes the totally transitive maps among transitive tree maps.

1. Introduction

The simplest type of a dynamical system consists of taking iterates of one continuous map $f : X \to X$ of a compact topological space X into itself. Given $x \in X$, the set $\{x, f(x), f^2(x), \ldots\}$ is called the *orbit of* x. This orbit is said to be a *periodic orbit of period* n if $f^n(x) = x$ and $f^i(x) \neq x$ for $i = 1, \ldots, n-1$. A *fixed point* is a periodic orbit with period $n = 1$. The set of periods of all periodic orbits of a map f will be denoted by $\mathrm{Per}(f)$.

The study of properties of orbits (periodicity, density, etc.) contributes to the knowledge of the behavior of the system. Systems with complicated behavior are usually called chaotic. Chaos is defined in many different ways throughout the literature. In [Devaney, 1986], f is said to be chaotic if it is *transitive*, its periodic points are dense and it has *sensitivity with respect to the initial conditions*. It is shown in [Banks *et al.*, 1992] that the last condition is a consequence of the first two when X is an infinite metric space. Finally, in [Blokh, 1987] it is proved that if X is a graph and f is a transitive map with at least one periodic point, then the set of periodic points of f is dense in X. Therefore, for maps of connected graphs with periodic points (i.e. those which are transitive and not conjugate to an irrational rotation of the circle), chaos in the sense of Devaney [1986] is equivalent to transitivity.

Other usual notions of chaos in the context of one-dimensional dynamics are:

(a) positive topological entropy,
(b) chaos in the sense of Li and Yorke [1975].

The three notions of chaos provide a stratification of maps according to their complexity. Indeed, for graph maps with periodic points, transitivity implies positive topological entropy [Blokh, 1995; Llibre & Misiurewicz, 1993], which in turn implies chaos in the sense of Li and Yorke (see [Llibre & Misiurewicz, 1993]).

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