Letters

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A NOTE ON THE TOTALLY TRANSITIVE GRAPH MAPS

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We study the totally transitive graph maps which have periodic points. In particular we show that the cofiniteness of the set of periods characterizes these maps among transitive graph maps.

1. Introduction

The aim of this paper is to generalize the main result of [Alsedà *et al.*, 1999b] to transitive graph maps. To this end we have to introduce some notation.

Let X be a topological space. A continuous map $f : X \to X$ is called (*topologically*) *transitive* if for every nonempty open subsets $U, V \subset X$ there exists $n \geq 1$ such that $f^n(U) \cap V \neq \emptyset$. In a connected compact metric space a map is transitive if and only if has a dense orbit. A transitive map is called *totally transitive* if f^s is transitive for all $s \geq 1$. Also, f is said to be *topologically mixing* if for every nonempty open subsets $U, V \subset X$ there exists $N \in \mathbb{N}$ such that $f^n(U) \cap V \neq \emptyset$ for each $n > N$.

The study of transitive maps on compact spaces is widely motivated. For example, the asymptotic dynamics of dissipative systems are the dynamics inherent in the attractors. Here, attractor means a compact invariant set containing a dense orbit (transitivity) and whose stable set (attracting basin) has a nonempty interior or, at least, positive Lebesgue measure. Due to the dissipation, the attractors have lower dimension than the phase space. Their dynamics often reduce to a one-dimensional one and, despite the low dimension, interesting questions arise. One-dimensional dynamics also appear when the system has an invariant foliation of codimension one. In this way, as it has been shown in [Williams, 1977], the iteration of a map on a graph imitates the behavior of a flow in a neighborhood of a hyperbolic attractor.

It is important to know if for a transitive map there exists some iterate which is not transitive as the following example shows. Let φ be the map defined on the interval $[0, 1]$ in the following way:

$$\varphi(x) = \begin{cases} \dfrac{1}{2} + 2x & \text{if } 0 \leq x \leq \dfrac{1}{4}\,, \\[2mm] \dfrac{3}{2} - 2x & \text{if } \dfrac{1}{4} \leq x \leq \dfrac{1}{2}\,, \\[2mm] 1 - x & \text{if } \dfrac{1}{2} \leq x \leq 1\,. \end{cases}$$

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