

A Survey on the Relation Between Transitivity and Dense Periodicity for Graph Maps

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Dedicated to Professor Alexander N. Sharkovsky on the occasion of his 65th Birthday.

This paper is an outline of the relation between the transitivity, the dense periodicity and the topological entropy of graph maps.

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The notion of topological transitivity, due to Birkhoff [13] (see also Ref. [12]), is very well motivated in Ref. [21] as follows: “... one may think of a real physical system, where a state is never given or measured exactly, but always up to a certain error. So instead of points one should study (small) open subsets of the phase space and describe how they move in that space. ... Intuitively, a topologically transitive map f has points which eventually move under iteration from one arbitrarily small neighborhood to any other. Consequently, the dynamical system cannot be broken down or decomposed into two subsystems (disjoint sets with nonempty interiors) which do not interact under f , i.e., are invariant under the map. ...” In particular, the notion of topological transitivity is a global one in the system. The above considerations lead in a natural way to the following definition:

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