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Transitivity and Dense Periodicity for Graph Maps

LL. ALSEDÀ[a,*], M.A. DEL RÍO[b,†] and J.A. RODRÍGUEZ[c,‡]

[a]*Departament de Matemàtiques, Edifici Cc, Universitat Autònoma de Barcelona, 08913 Cerdanyola del Vallès, Barcelona, Spain;* [b]*Departamento de Análise Matemática, Universidade de Santiago de Compostela, 15706 Santiago de Compostela, Spain;* [c]*Departamento de Matemáticas, Universidad de Oviedo, 33007 Oviedo, Spain*

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In 1984, Blokh proved [A. M. Blockh, On transitive mappings of one-dimensional branched manifolds, Differential-Difference Equations and Problems of Mathematical physics (Russian), Akad. Nauk Ukrain. SSR Inst. Mat., Kiev, **131**, pp. 3–9, 1984] that any topologically transitive continuous map from a graph into itself which has periodic points has a dense set of periodic points and has positive topological entropy (in this proof a crucial role is played by the specification property, which implies these two statements). Also, he characterized the topologically transitive continuous graph maps without periodic points. Unfortunately, this clever paper is only available in Russian (except for a translation to English of the statements of the theorems without proofs—see [A. M. Blockh, The connection between entropy and transitivity for one-dimensional mappings, *Uspekhi Mat. Nauk*, **42**(5(257)) (1987), pp. 209–210]).

The present paper is an update on the basic properties of the topologically transitive graph maps with special emphasis on the density of the set of periodic points. In particular, we give full proofs of the aforementioned Blokh's results.

Keywords: Topological transitivity; Graph maps; Density of the set of periods; Topological entropy

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INTRODUCTION

According to Gottschalk and Hedlund [24] the notion of topological transitivity is due to Birkhoff [14] (see also Ref. [13]). The following excellent motivation for this notion can be found in Ref. [25]: "... one may think of a real physical system, where a state is never given or measured exactly, but always up to a certain error. So instead of points one should study (small) open subsets of the phase space and describe how they move in that space.... Intuitively, a topologically transitive map f has points which eventually move under iteration from one arbitrarily small neighborhood to any other. Consequently, the dynamical system cannot be broken down or decomposed into two subsystems (disjoint sets with nonempty interiors) which do not interact under f, i.e. are invariant under the map ...." In particular,

*Corresponding author. E-mail: alseda@mat.uab.es
†E-mail: mdelrio@zmat.usc.es
‡E-mail: chachi@pinon.ccu.uniovi.es

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