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ON THE SET OF PERIODS OF SIGMA MAPS OF DEGREE 1

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Abstract. We study the set of periods of degree 1 continuous maps from σ into itself, where σ denotes the space shaped like the letter σ (i.e., a segment attached to a circle by one of its endpoints). Since the maps under consideration have degree 1, the rotation theory can be used. We show that, when the interior of the rotation interval contains an integer, then the set of periods (of periodic points of any rotation number) is the set of all integers except maybe 1 or 2. We exhibit degree 1 σ-maps f whose set of periods is a combination of the set of periods of a degree 1 circle map and the set of periods of a 3-star (that is, a space shaped like the letter Y). Moreover, we study the set of periods forced by periodic orbits that do not intersect the circuit of σ; in particular, when there exists such a periodic orbit whose diameter (in the covering space) is at least 1, then there exist periodic points of all periods.

1. Introduction.

In this paper we study the set of periods of continuous maps from the space σ to itself, where the space σ consists of a circle with a segment attached to it at one of the segment's endpoints. Our results continue the progression of results which began with Sharkovskii's Theorem on the characterization of the sets of periods of periodic points of continuous interval maps [20, 21] and continued with the study of the periods of maps of the circle [14, 13, 19], trees [2, 3, 5, 4, 12, 6, 11] and other graphs [16, 17].

A full characterization of the sets of periods for continuous self maps of the graph σ having the branching fixed is given in [16]. Our goal is to extend this result to the general case. The most natural approach is to follow the strategy used in the circle case which consists in dividing the problem according to the degree of the map [14, 13, 19]. The cases considered for the circle are degree different from $\{-1, 0, 1\}$, and separately the cases of degree 0, -1 and 1. A characterization of the

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