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DIVISION FOR STAR MAPS WITH THE BRANCHING POINT FIXED

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Abstract. We extend the notion of division given for interval maps (see [**10**]) to the n-star and study the set of periods of star maps such that all their periodic orbits with period larger than one have a division. As a consequence of this result we get some conditions characterizing the star maps with zero topological entropy.

1. Introduction

The **n-star** is the subspace of the plane which is most easily described as the set of all complex numbers z such that z^n is in the unit interval $[0, 1]$. We shall denote the n-star by $\mathbf{X}_n$. We shall also use the notation $\mathcal{X}_n$ to denote the class of all continuous maps from $\mathbf{X}_n$ to itself such that $f(0) = 0$.

We note that the 1-star and the 2-star are homeomorphic to a closed interval of the real line. Thus, in what follows, when talking about $\mathbf{X}_n$ or $\mathcal{X}_n$ we shall always assume that $n \geq 2$.

As usual, if $f \in \mathcal{X}_n$ we shall write f^k to denote $f \circ f \circ \cdots \circ f$ (k times). A point $x \in \mathbf{X}_n$ such that $f^k(x) = x$ but $f^j(x) \neq x$ for $j = 1, 2, \dots, k-1$ will be called a **periodic point of f of period k**. If x is a periodic point of f of period m then the set $\{f^k(x) : k > 0\}$ will be called a **periodic orbit** of f of period m (of course it has cardinality m).

The set of periods of all periodic points of a map $f \in \mathcal{X}_n$ will be denoted by $\operatorname{Per}(f)$.

In this paper we extend the notion of division given for interval maps (see [**10**]) and for maps from $\mathcal{X}_3$ (see [**4**]) to the n-star and we study the set of periods of maps from $\mathcal{X}_n$ such that all their periodic orbits have a division. As a consequence of this result we get some conditions characterizing the maps from $\mathcal{X}_n$ with zero topological entropy.

We start by fixing the notion of division. The components of $\mathbf{X}_n \setminus \{0\}$ will be called branches.

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