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No division and the set of periods for tree maps

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Abstract. We extend the notion of no division for star maps to tree maps and prove that the set of periods of a tree map is cofinite if there exists some periodic orbit of the given map with period larger than one having no division. Using this result we obtain some simple proofs of known results about the set of periods of a tree map. Also we show that this set is a union of initial segments of a finite number of linear orderings which depend only on the given tree minus a finite subset of $\mathbb{N}$.

1. *Introduction*

In recent years there has been a growing interest in the study of the dynamical behavior of continuous maps from a *graph*, i.e. a one-dimensional connected branched compact manifold, into itself (see for instance [**1**]–[**10**], [**12**], [**13**] and [**16**]). In this study an important problem is to determine the set of periods of all periodic orbits of a continuous map from a graph into itself. This set provides a lot of information on the dynamical properties of the system.

In this article we shall deal with this problem in the particular case of a *tree*, i.e. a graph without cycles. For simplicity, a continuous map from a tree (resp. interval) into itself will be called a *tree map* (resp. *interval map*). The first remarkable result concerning the set of periods of a given tree was reported by Sharkovskiĭ who gave a complete characterization of the set of periods for interval maps. This result is the following so-called Sharkovskiĭ's theorem. To state it, first we have to introduce the Sharkovskiĭ's ordering in the set $\mathbb{N} \cup \{2^\infty\}$ as follows:

$$3 \,{}_s\!\!> 5 \,{}_s\!\!> 7 \,{}_s\!\!> \dots \,{}_s\!\!> 2 \cdot 3 \,{}_s\!\!> 2 \cdot 5 \,{}_s\!\!> 2 \cdot 7 \,{}_s\!\!> \dots \,{}_s\!\!> 4 \cdot 3 \,{}_s\!\!> 4 \cdot 5 \,{}_s\!\!> 4 \cdot 7 \,{}_s\!\!> \dots$$
$${}_s\!\!> \dots \,{}_s\!\!> 2^n \cdot 3 \,{}_s\!\!> 2^n \cdot 5 \,{}_s\!\!> 2^n \cdot 7 \,{}_s\!\!> \dots \,{}_s\!\!> 2^\infty \,{}_s\!\!> \dots \,{}_s\!\!> 2^n \,{}_s\!\!> \dots \,{}_s\!\!> 16$$
$${}_s\!\!> 8 \,{}_s\!\!> 4 \,{}_s\!\!> 2 \,{}_s\!\!> 1.$$

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