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Existence of non-trivial limit cycles in Abel equations with symmetries

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ABSTRACT

We study the periodic solutions of the generalized Abel equation $x' = a_1A_1(t)x^{n_1} + a_2A_2(t)x^{n_2} + a_3A_3(t)x^{n_3}$, where $n_1, n_2, n_3 > 1$ are distinct integers, $a_1, a_2, a_3 \in \mathbb{R}$, and A_1, A_2, A_3 are 2π-periodic analytic functions such that $A_1(t)\sin t, A_2(t)\cos t, A_3(t)\sin t\cos t$ are π-periodic positive even functions.

When $(n_3 - n_1)(n_3 - n_2) < 0$ we prove that the equation has no non-trivial (different from zero) limit cycle for any value of the parameters a_1, a_2, a_3.

When $(n_3 - n_1)(n_3 - n_2) > 0$ we obtain under additional conditions the existence of non-trivial limit cycles. In particular, we obtain limit cycles not detected by Abelian integrals.

1. Introduction and main result

The number of limit cycles (periodic solutions isolated in the set of periodic solutions) of generalized Abel equations

$$x' = c_1(t)x + c_2(t)x^2 + \cdots + c_n(t)x^n, \tag{1.1}$$

where $c_1, c_2, \ldots, c_n$ are 2π-periodic functions has been intensively studied due to its relation to Hilbert's 16th problem. This famous unsolved problem deals with the number and location of limit cycles of the planar system

$$x' = P(x, y), \qquad y' = Q(x, y),$$

where $P(x, y), Q(x, y)$ are nth-degree polynomials of x and y.

When P, Q are quadratic, this problem is equivalent to the determination of limit cycles of

$$x' = c_1x + c_2(t)x^2 + c_3(t)x^3,$$

where $c_1 \in \mathbb{R}$ and $c_2(t), c_3(t)$ are trigonometric polynomials [1]. Some higher degree planar systems, in particular rigid systems, can also be reduced to generalized Abel equations [2–4].

Even in the case $n = 3$, the number of limit cycles of (1.1) is not bounded [1]. Thus, to obtain upper bounds for the number of limit cycles one must ask for some conditions to be set on the coefficients $c_1(t), c_2(t), \ldots, c_n(t)$. These additional conditions are that some of $c_k(t)$ have definite sign [5–8], that $c_n(t) \equiv 1$ and the rest of the coefficients are bounded by a certain constant [9,10], or that there exists a linear combination of some of the coefficients with definite sign [11–13].

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