A NOTE ON THE DIFFERENTIABILITY OF A FAMILY OF NON-GRAVITATIONAL CENTRAL FORCE CONTINUOUS POTENTIALS IN DIMENSION ONE

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Abstract. The present paper offers an alternative point of view of the study for the motion a system of two particles which will always move in some fixed line, we take $\mathbb{R}$ for the position space. If we fix the center of mass at the origin, so the system reduces to that of a single particle of unit mass in a central force field. We take the potential energy function $U(x) = |x|^\beta$, where x is the position of the single particle and β some positive real number.

1. Introduction

In 1981, R. McGehee [2] investigated geometrically the regularization of binary collisions of classical particle systems with non-gravitational interactions. R. McGehee considered the motion near a collision of a particle in the vector field given by the homogeneous potential $U(x) = -|x|^{-\alpha}$, where $x \in \mathbb{R}^2$ is the position of a single particle and α is a positive real number. McGehee showed by appropriate coordinate transformations that the singularity corresponding to a double collision ($x = 0$) is blown up to a collision manifold, after that the time variable is rescaled appropriately, and finally the vector field is extended smoothly to this manifold. He noted that there exits a bifurcation at $\alpha = 2$.

More recently, Xia and Jardón-Kojakhmetov [3] investigated the topological structure of the same system as α varies along the entire real line $\mathbb{R}$. This study recovers the results of the previous reference and tries to extend the analysis to $\alpha \leq 0$. They use the McGehee techniques without considering that for $\alpha \leq 0$ the potential $U(x)$ is

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