Celestial Mechanics and Dynamical Astronomy (2006) 95:173–200
DOI 10.1007/s10569-006-9018-9

ORIGINAL ARTICLE

On final evolutions in the restricted planar parabolic three-body problem

Martha Alvarez · Josep Maria Cors · Joaquin Delgado

Received: 15 November 2005 / Revised: 13 March 2006 /
Accepted: 28 March 2006 / Published online: 15 August 2006

Abstract In this paper, we prove the existence of special type of motions in the restricted planar parabolic three-body problem, of the type exchange, emission–capture, and emission–escape with close passages to collinear and equilateral triangle configuration, among others. The proof is based on a gradient-like property of the Jacobian function when equations of motion are written in a rotating–pulsating reference frame, and the extended phase space is compactified in the time direction. Thus a phase space diffeomorphic to $[-\pi/2, \pi/2] \times \mathbb{C}\backslash\{-\mu_1, \mu_2\} \times \mathbb{C}$ -coordinates (θ, ζ, ζ') is obtained with the boundary manifolds $\theta = \pm\pi/2$ corresponding to escapes of the binaries when time tends to $\pm\infty$. It is shown there exists exactly five critical points on each boundary, corresponding to classic homographic solutions. The connections of the invariant manifolds associated to the collinear configurations, and stable/unstable sets associated to binary collision on the boundary manifolds, are obtained for arbitrary masses of the primaries. For equal masses extra connections are obtained, which include equilateral configurations. Based on the gradient-like property, a geometric criterion for capture is proposed and is compared with a criterion introduced by Merman (1953b) in the fifties, and an example studied numerically by Kocina (1954).

Keywords Restricted three-body problem · Final evolutions · Heteroclinic orbits

M. Alvarez · J. Delgado
Departamento de Matemáticas, Universidad Autónoma Metropolitana, Iztapalapa,
A.P. 55-534, Iztapalapa 09340, México, D.F., México
e-mails: mar@xanum.uam.mx; jdf@xanum.uam.mx

J. M. Cors (✉)
Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya,
08240 Manresa, Spain
e-mail: cors@eupm.upc.es