PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 133, Number 2, Pages 529–536
S 0002-9939(04)07590-2
Article electronically published on August 26, 2004

THE NUMBER OF PLANAR CENTRAL CONFIGURATIONS FOR THE 4–BODY PROBLEM IS FINITE WHEN 3 MASS POSITIONS ARE FIXED

MARTHA ALVAREZ, MONTSERRAT CORBERA, JOAQUIN DELGADO, AND JAUME LLIBRE

(Communicated by Carmen C. Chicone)

ABSTRACT. In the n–body problem a central configuration is formed when the position vector of each particle with respect to the center of mass is a common scalar multiple of its acceleration vector. Lindstrom showed for $n = 3$ and for $n > 4$ that if $n - 1$ masses are located at fixed points in the plane, then there are only a finite number of ways to position the remaining nth mass in such a way that they define a central configuration. Lindstrom leaves open the case $n = 4$. In this paper we prove the case $n = 4$ using as variables the mutual distances between the particles.

1. INTRODUCTION

For the n–body problem a configuration of the system of n particles is central if the acceleration of each mass is proportional to its position relative to the center of mass of the system.

Central configurations play an important role in the n–body problem of celestial mechanics. For instance, they allow one to obtain the homographic solutions (the unique solutions of the n–body problem that we can describe explicitly) [13], central configurations play a main role in the topological changes of the integral manifolds [11], and they are the limiting configurations for colliding particles [7] or parabolic escape [10].

Some interesting results for the planar central configurations of the n–body problem have been achieved, but the problem is far from solved. The sixth problem of Smale's list presenting his challenging mathematical problems for the twenty–first century [12], cites Wintner's question of whether, for a given set of n positive masses, the number of nonequivalent (modulus rotations and rescalings) planar central configurations is finite.

In [5] Lindstrom formulated a program of research as follows: Given n positive masses $m_1, m_2, \dots, m_n$ and for any $k = 1, 2, \dots, n-2$, given their $n-k$ positions in the plane, to determine whether there are only a finite number of ways to position the remaining k particles in a manner that defines a central configuration. For given

Received by the editors November 12, 2002 and, in revised form, February 6, 2003 and July 16, 2003.
2000 *Mathematics Subject Classification.* Primary 70F07; Secondary 70F15.
Key words and phrases. 4–body problem, central configurations.