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ON THE SPATIAL CENTRAL CONFIGURATIONS OF THE 5–BODY PROBLEM AND THEIR BIFURCATIONS

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Abstract. Central configurations provide special solutions of the general n–body problem. Using the mutual distances between the n bodies as coordinates we study the bifurcations of the spatial central configurations of the 5–body problem going from the problem with equals masses to the 1+4– body problem which has one large mass and four infinitesimal equal masses. This study is made by giving a computer–aided proof.

1. **Introduction and statement of results.** Central configurations are important in the n–body problem because they allow to obtain the homographic solutions, those solutions of the n–body problem that can be described explicitly [22]. They play a main role in the topological changes of the integral manifolds [20], and they are the limiting configurations either for colliding particles [10] or for parabolic total escape [18].

We consider n particles of positive masses m_i, $i = 1, \ldots, n$ moving in $\mathbb{R}^3$ under their mutual Newtonian gravitational attraction. Let $\mathbf{q}_i$ be the position vector of the ith particle relative to the center of mass, by Newton's law, the equations of motion are

$$m_i \ddot{\mathbf{q}}_i = \frac{\partial U}{\partial \mathbf{q}_i},$$

where the potential is

$$U = \sum_{1 \le i < j \le n} \frac{G m_i m_j}{\|\mathbf{q}_j - \mathbf{q}_i\|}.$$

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