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Local behavior of planar analytic vector fields via integrability ☆

María Jesús Álvarez [a], Antoni Ferragut [b,*]

[a] Departament de Matemàtiques i Informàtica, Universitat de les Illes Balears, Edifici Anselm Turmeda, Ctra. de Valldemossa, km 7.5, 07122 Palma, Illes Balears, Spain
[b] Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, Av. Diagonal, 647, 08028 Barcelona, Catalonia, Spain

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A.F. wants to dedicate this paper to the memory of his father

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ABSTRACT

We present an algorithm to study the local behavior of singular points of planar analytic vector fields having a first integral which is a quotient of analytic functions. The algorithm is based on the blow-up method. It emphasizes the curves passing through the singular points and avoids the computation of the desingularized systems. Vector fields having a rational first integral are a particular case.

1. Introduction

A *real planar analytic vector field* is a vector field defined on $\mathbb{R}^2$ of the form

$$X(x,y) = P(x,y)\frac{\partial}{\partial x} + Q(x,y)\frac{\partial}{\partial y}, \tag{1}$$

where P and Q are coprime analytic functions. We refer to the vector field (1) or equivalently to its associated *planar analytic differential system*

$$\dot{x} = P(x,y), \qquad \dot{y} = Q(x,y). \tag{2}$$

Let $m = \min\{m_P, m_Q\}$ be the *multiplicity of the vector field* (1) *at the origin*, where m_P and m_Q are the multiplicities of P and Q at the origin, respectively.

The study of the topological behavior of the solutions of a planar differential system in a neighborhood of a singular point is one of the main unsolved problems in the qualitative theory of differential systems. Concerning the singular points having at least one eigenvalue different from zero, the problem is solved except for the center-focus case. Regarding the degenerate singular points, with both eigenvalues of the jacobian matrix at the point equal to zero, the situation is more complicated. The topology around a non-monodromic singular point can be much richer. The Andreev Theorem (see [3]) classifies the nilpotent singular points (degenerate singular points whose associated jacobian matrix is not identically zero) except the center-focus case. If the jacobian matrix is identically null the problem is open. In this case, the only possibility is studying each degenerate point case by case. The main technique which is used to perform the study of this kind of points

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* Corresponding author.
E-mail addresses: chus.alvarez@uib.es (M.J. Álvarez), antoni.ferragut@upc.edu (A. Ferragut).

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