

## A SURVEY ON THE BLOW UP TECHNIQUE

MARÍA JESÚS ÁLVAREZ\*

Departament de Matemàtiques i Informàtica, Universitat de les Illes Balears, Crtra. Valldemossa, km 7.5, 07122 Palma, Spain chus.alvarez@uib.es

ANTONI FERRAGUT<sup>†</sup>

Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, Av. Diagonal, 647, 08028 Barcelona, Catalunya, Spain antoni.ferragut@upc.edu

## XAVIER JARQUE<sup>‡</sup>

Departament d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, Av. Països Catalans, 26, 43007 Tarragona, Catalunya, Spain xavier.jarque@ub.edu

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The blow up technique is widely used in desingularization of degenerate singular points of planar vector fields. In this survey, we give an overview of the different types of blow up and we illustrate them with many examples for better understanding. Moreover, we introduce a new generalization of the classical blow up.

Keywords: Planar differential system; degenerate singular point; blow up.

## 1. Introduction

The study of the topological behavior of the solutions of a real planar vector field  $X = P\partial_x + Q\partial_y$ in a neighborhood of a singular point is one of the main unsolved problems in the qualitative theory of differential systems. Concerning the simple singular points (where both eigenvalues of the Jacobian matrix at the singular point are different from zero) the Hartman–Grobman Theorem completely classifies them (except the center-focus points). The semi-simple points (with one of the eigenvalues equal to zero) are also classified (see [Andronov *et al.*, 1973]). Precisely, the local phase portrait in these two cases is either monodromic (thus a focus or a center) or a saddle, or a node, or a saddle-node.

Regarding the degenerate singular points, with both eigenvalues of the Jacobian matrix at the point equal to zero, the situation is far more complicated. The topology around a nonmonodromic singular point can be much richer. The Andreev Theorem

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