International Journal of Bifurcation and Chaos

On the homoclinic orbits of the Lü system

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Received (to be inserted by publisher)

In this paper the existence of homoclinic orbits of the equilibrium point $(0, 0, 0)$ is demonstrated in the case of the of Lü system for parameter values did not report by G. A. Leonov. In addition, some simulations are shown that agree with our theoretical analysis.

Keywords: Lü system, chaotic, homoclinic orbit, bifurcation

1. Introduction

In this paper we consider the Lü system [Lü & Chen , 2002]

$$\begin{aligned} \dot{x} &= a(y - x), \\ \dot{y} &= cy - xz, \\ \dot{z} &= -bz + xy \end{aligned} \tag{1}$$

where $a, b, c \in \mathbb{R}^+$ are parameters.

For the parameter set given by $a = 36$, $b = 3$ and $c = 20$ this system (1) is a chaotic system which bridges the gap between the Lorenz and the Chen attractors [Lü & Chen , 2002]. They all have the same symmetry, stability of equilibrium points, and similar bifurcations and topological structures, in among other things [Lü & et. al. , 2002].

In the study of the chaotic systems a particular role is played by the homoclinic orbits because the existence of these suggests the existence of chaotic dynamics, see [Guckenheimer & Holmes , 1983]. The system (1) has a saddle point at $(0, 0, 0)$, for all positive parameter values. This paper concerned with the problem of finding conditions to ensure the existence of homoclinic orbits (joins the saddle equilibrium point to itself) of Lü system at origin.

Since the introduction of the Lü system, several articles related to this system of equations have been published, see for instance [Leonov, 2012], [Leonov , 2013], [Leonov & Kuznetsov , 2015], [Leonov , 2016] and references therein. One central aspect concerns to the existence of homoclinic orbits related to $(0, 0, 0)$.