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A new uniqueness criterion for the number of periodic orbits of Abel equations

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Abstract

A solution of the Abel equation $\dot{x} = A(t)x^3 + B(t)x^2$ such that $x(0) = x(1)$ is called a periodic orbit of the equation. Our main result proves that if there exist two real numbers a and b such that the function $aA(t) + bB(t)$ is not identically zero, and does not change sign in [0, 1] then the Abel differential equation has at most one non-zero periodic orbit. Furthermore, when this periodic orbit exists, it is hyperbolic. This result extends the known criteria about the Abel equation that only refer to the cases where either $A(t) \not\equiv 0$ or $B(t) \not\equiv 0$ does not change sign. We apply this new criterion to study the number of periodic solutions of two simple cases of Abel equations: the one where the functions $A(t)$ and $B(t)$ are 1-periodic trigonometric polynomials of degree one and the case where these two functions are polynomials with three monomials. Finally, we give an upper bound for the number of isolated periodic orbits of the general Abel equation $\dot{x} = A(t)x^3 + B(t)x^2 + C(t)x$, when $A(t)$, $B(t)$ and $C(t)$ satisfy adequate conditions.

Keywords: Abel equation; Periodic orbit

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