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Research paper

On the equilateral pentagonal central configurations

M. Alvarez-Ramírez [a,*], A. Gasull [b,c], J. Llibre [b]

[a] Departamento de Matemáticas, UAM–Iztapalapa, 09340 Iztapalapa, Mexico City, Mexico
[b] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
[c] Centre de Recerca Matemàtica, Edifici Cc, Campus de Bellaterra, 08193 Cerdanyola del Vallès (Barcelona), Spain

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ABSTRACT

An equilateral pentagon is a polygon in the plane with five sides of equal length. In this paper we classify the central configurations of the 5-body problem having the five bodies at the vertices of an equilateral pentagon with an axis of symmetry. We prove that there are two unique classes of such equilateral pentagons providing central configurations, one concave equilateral pentagon and one convex equilateral pentagon, the regular one. A key point of our proof is the use of rational parameterizations to transform the corresponding equations, which involve square roots, into polynomial equations.

1. Introduction and statement of the result

The Newtonian planar 5-body problem describes the dynamics of five point particles of positive masses m_i at positions $\mathbf{q}_i \in \mathbb{R}^2$ moving according to the Newton's laws under their mutual gravitational forces. The equations of motion of this 5-body problem are

$$m_i\ddot{\mathbf{q}}_i = -\sum_{j=1,j\neq i}^{5} Gm_im_j\frac{\mathbf{q}_i-\mathbf{q}_j}{r_{ij}^3}, \qquad 1\le i\le 5,$$

where $r_{ij} = |\mathbf{q}_i - \mathbf{q}_j|$ is the mutual distances between the masses m_i and m_j, and G is the gravitational constant. We take conveniently the time unit so that $G = 1$.

The configuration space is defined by

$$\mathcal{E} = \{\mathbf{q} = (\mathbf{q}_1, \dots, \mathbf{q}_5) \in (\mathbb{R}^2)^5 : \mathbf{q}_i \neq \mathbf{q}_j, \quad i \neq j\}.$$

The configuration $\mathbf{q} = (\mathbf{q}_1, \dots, \mathbf{q}_5)$ is called *central* if the position vector of each body with respect to the center of mass is proportional to the corresponding acceleration vector. In other words, if there exists a positive constant λ such that

$$\ddot{\mathbf{q}}_i = \lambda(\mathbf{q}_i - \mathbf{c}_m), \qquad i = 1, \dots, 5,$$

where $\mathbf{c}_m = (m_1\mathbf{q}_1 + \cdots + m_5\mathbf{q}_5)/M$ and $M = m_1 + \cdots + m_5$, being $\mathbf{c}_m$ and M the center of mass of the five bodies and the total mass, respectively. Hence a given configuration $(\mathbf{q}_1, \dots, \mathbf{q}_5) \in \mathcal{E}$ of the 5-body problem with positive masses $m_1, \dots, m_5$, is central if there exists a λ such that $(\lambda, \mathbf{q}_1, \dots, \mathbf{q}_5)$ is a solution of the system

$$\sum_{j=1,j\neq i}^{5} m_j\frac{\mathbf{q}_i-\mathbf{q}_j}{r_{ij}^3} = \lambda(\mathbf{q}_i - \mathbf{c}_m), \qquad 1\le i\le 5. \tag{1}$$

* Corresponding author.
E-mail addresses: mar@xanum.uam.mx (M. Alvarez-Ramírez), gasull@mat.uab.cat (A. Gasull), jllibre@mat.uab.cat (J. Llibre).

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