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Topological classification of polynomial complex differential equations with all the critical points of centre type

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Dedicated to Robert Devaney on the occasion of his 60th birthday

In this paper, we study the global phase portrait of complex polynomial differential equations of degree n of the form $\dot{z} = f(z)$, having all their critical points of centre type. We give the exact number of topologically different phase portraits on the Poincaré disk when $n \leq 6$ and in the remaining cases, an upper bound for this number in terms of n.

Keywords: ordinary differential equation; phase portrait; critical point; centre; planar graph; separatrix configuration

AMS Subject Classification: 37C10; 34C05; 32A10; 34A34

1. Introduction and main result

In this work, we deal with the equation

$$\dot{z} := \frac{dz}{dt} = f(z), \quad t \in \mathbb{R}, \quad z \in \mathbb{C}, \tag{1}$$

where f is a complex polynomial of degree $n \geq 1$.

Many characteristics of equation (1) are well-known. It presents only three types of finite simple critical points (focus, centre and node) all of index $+1$ and the centres are always isochronous. Moreover, this type of equations cannot have limit cycles, see for instance [5,11,15,16]. Regarding the points at infinity in the Poincaré compactification, it is proved that they have exactly $n - 1$ couples of saddle points (see [1,11]).

In the previous paper [1], the authors explored the relationship between the geometric distribution of the critical points and their type, paying special attention to the case in which the critical points were centres.

In this paper, we are going to assume that all the critical points are centres and we will be concerned with the global phase portrait of equation (1) on the Poincaré disk. By the results of [21] (that we recall in Theorem 2.2), it turns out that this problem is equivalent to a separatrix configurations problem. In our context, the only separatrices will be the ones of the saddle points at infinity. The problem of studying the number of different phase

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