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Global behaviour of the period function of the sum of two quasi-homogeneous vector fields

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ABSTRACT

We study the global behaviour of the period function on the period annulus of degenerate centres for two families of planar polynomial vector fields. These families are the quasi-homogeneous vector fields and the vector fields given by the sum of two quasi-homogeneous Hamiltonian ones. In the first case we prove that the period function is globally decreasing, extending previous results that deal either with the Hamiltonian quasi-homogeneous case or with the general homogeneous situation. In the second family, and after adding some more additional hypotheses, we show that the period function of the origin is either decreasing or has at most one critical period and that both possibilities may happen. This result also extends some previous results that deal with the situation where both vector fields are homogeneous and the origin is a non-degenerate centre.

1. Introduction and main results

A planar polynomial vector field $X(x,y) = (P(x,y), Q(x,y))$ is called (p,q) quasi-homogeneous of quasi-degree n if there exist $p, q, n \in \mathbb{N}$ such that

$$P(\lambda^p x, \lambda^q y) = \lambda^{n+p-1} P(x,y), \quad Q(\lambda^p x, \lambda^q y) = \lambda^{n+q-1} Q(x,y),$$

for all $\lambda \in \mathbb{R}$. It is not restrictive to take p and q coprime. The numbers p and q are usually called weights. It is well known that its associated differential equation

$$\begin{cases} \dot{x} = P(x,y), \\ \dot{y} = Q(x,y), \end{cases}$$

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