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Lower bounds for the number of limit cycles of trigonometric Abel equations

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Abstract

We consider the Abel equation $\dot{x} = A(t)x^3 + B(t)x^2$, where $A(t)$ and $B(t)$ are trigonometric polynomials of degree n and m, respectively, and we give lower bounds for its number of isolated periodic orbits for some values of n and m. These lower bounds are obtained by two different methods: the study of the perturbations of some Abel equations having a continuum of periodic orbits and the Hopf-type bifurcation of periodic orbits from the solution $x = 0$.
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1. Introduction and main results

In this paper we deal with the Abel equation

$$\frac{dx}{dt} = A(t)x^3 + B(t)x^2, \tag{1.1}$$

where $A(t)$ and $B(t)$ are 2π-trigonometric polynomials of degrees n and m, respectively. A solution of the previous equation satisfying $x(0) = x(2\pi)$ is called a periodic orbit. It is easy to prove from the results of [7] that if either $A(t)$ or $B(t)$ does not change sign, the previous equation has at most two isolated periodic orbits. The same result also holds if there exist two real numbers a and b such that the function $aA(t) + bB(t)$ does not change sign, see [1]. But if none of the above mentioned conditions is satisfied, it is not known how to bound the number of periodic orbits that Eq. (1.1) can have. It is neither known how to obtain this bound depending only on the degrees of $A(t)$ and $B(t)$, see [9].

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