# Limit cycles for a class of quintic $\mathbb{Z}_6$-equivariant systems without infinite critical points

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**Abstract**

We analyze the dynamics of a class of $\mathbb{Z}_6$-equivariant systems of the form $\dot{z} = pz^2\bar{z} + sz^3\bar{z}^2 - \bar{z}^5$, where $z$ is complex, the time $t$ is real, while $p$ and $s$ are complex parameters. This study is the natural continuation of a previous work (M.J. Álvarez, A. Gasull, R. Prohens, Proc. Am. Math. Soc. **136**, (2008), 1035–1043) on the normal form of $\mathbb{Z}_4$-equivariant systems. Our study uses the reduction of the equation to an Abel one, and provide criteria for proving in some cases uniqueness and hyperbolicity of the limit cycle surrounding either 1, 7 or 13 critical points, the origin being always one of these points.

## 1 Introduction and main results

Hilbert $XVI^{th}$ problem represents one of the open question in mathematics and it has produced an impressive amount of publications throughout the last century. The study of this problem in the context of equivariant dynamical systems is a relatively new branch of analysis and is based on the development within the last twenty years of the theory of Golubitsky, Stewart and Schaeffer in [9, 10]. Other authors [6] have specifically considered this theory when studying the limit cycles and related phenomena in systems with symmetry. Roughly speaking the

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