Arch. Rational Mech. Anal. 156 (2001) 317–357
Digital Object Identifier (DOI) 10.1007/s002050100116

Heteroclinic Orbits and Bernoulli Shift for the Elliptic Collision Restricted Three-Body Problem

MARTHA ALVAREZ & JAUME LLIBRE

Communicated by P. H. RABINOWITZ

Abstract

We consider two mass points of masses $m_1 = m_2 = \frac{1}{2}$ moving under Newton's law of gravitational attraction in a collision elliptic orbit while their centre of mass is at rest. A third mass point of mass $m_3 \approx 0$, moves on the straight line L, perpendicular to the line of motion of the first two mass points and passing through their centre of mass. Since $m_3 \approx 0$, the motion of the masses m_1 and m_2 is not affected by the third mass, and from the symmetry of the motion it is clear that m_3 will remain on the line L. So the three masses form an isosceles triangle whose size changes with the time. The elliptic collision restricted isosceles three-body problem consists in describing the motion of m_3.

In this paper we show the existence of a Bernoulli shift as a subsystem of the Poincaré map defined near a loop formed by two heteroclinic solutions associated with two periodic orbits at infinity. Symbolic dynamics techniques are used to show the existence of a large class of different motions for the infinitesimal body.

1. Introduction

SITNIKOV [16] showed the possibility of the existence of oscillatory motions for the elliptic non-collision restricted isosceles three-body problem. This problem is now called the *Sitnikov problem*. ALEKSEEV [1,2] (see also MOSER [13]) proved the existence of such a motion by using some homoclinic or heteroclinic orbits. We extended all the dynamics found in the elliptic non-collision restricted isosceles three-body problem to the collision problem. These two problems are very different due to the existence of the triple collision in the second problem.

We have two mass points with equal masses, $m_1 = m_2$ (called *primaries*), moving under Newton's law of gravitational attraction in a collision elliptic orbit while their centre of mass is at rest. We consider a third mass point of mass $m_3 \approx 0$, moving on the straight line L (z-axis) perpendicular to the line of motion (x-axis)