## THE SYMMETRIC CENTRAL CONFIGURATIONS OF THE 4–BODY PROBLEM WITH MASSES

 $m_1 = m_2 \neq m_3 = m_4$ 

MARTHA ALVAREZ–RAMÍREZ<sup>1</sup> AND JAUME LLIBRE<sup>2</sup>

ABSTRACT. We characterize the planar central configurations of the 4body problem with masses  $m_1 = m_2 \neq m_3 = m_4$  which have an axis of symmetry.

It is known that this problem has exactly two classes of convex central configurations, one with the shape of a rhombus and the other with the shape of an isosceles trapezoid.

We show that this 4-body problem also has exactly two classes of concave central configurations with the shape of a kite, this proof is assisted by computer.

## 1. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

The main problem of the classical Celestial Mechanics is the *n*-body problem; i.e. the description of the motion of *n* particles of positive masses under their mutual Newtonian gravitational forces. This problem is completely solved only when n = 2, and for n > 2 there are only few partial results. In this paper all the results commented or proved will be on positive masses, and we do not mention this again.

Central configurations are initial positions of the n bodies where the position and the acceleration vector of each particle with respect to the center of mass are proportional, with the same constant of proportionality for all the n particles. Central configurations started to be studied in the second part of the 18th century, there is an extensive literature concerning these solutions. For a classical background, see the sections on central configurations in the books of Wintner [28] and Hagihara [10]. For a modern background see, for instance, the papers of Saari [24], McCord [18], Palmore [22], Schmidt [25], Xia [29], ...

One of the reasons why central configurations are important is that they allow to obtain the unique explicit solutions in function of the time of the n-body problem known until now, the *homographic solutions* for which the



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