HJELMSLEV QUADRILATERAL CENTRAL CONFIGURATIONS

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ABSTRACT. A Hjelmslev quadrilateral is a quadrilateral with two right angles at opposite vertices. We classify all planar four–body central configurations where the four bodies are at the vertices of a Hjelmslev quadrilateral. We show that in the positive mass space (m_1, m_2, m_3, m_4), taking the unit of mass equal to m_1, the set of Hjelmslev quadrilateral central configurations of the four–body problem is an open arc. When the masses tend to the endpoints of this arc three of the masses of the Hjelmslev quadrilateral central configurations tend to an equilateral central configuration of the three–body problem, and the fourth remainder mass tends to zero.

1. Introduction and statement of the main results

The *n–body problem*, i.e. the description of the motion of n particles of positive masses under their mutual Newtonian gravitational forces, is the main problem of the classical Celestial Mechanics. Only the 2–body problem is completely solved, and for $n > 2$ there are only few partial results on the dynamics of the n–body problem.

In $\mathbb{R}^2$ the equations of motion of the n–body problem are

$$\ddot{x}_i = \sum_{j=1,\ j\neq i}^{n} \frac{m_j(x_j - x_i)}{r_{ij}^3}, \quad \text{for} \quad i = 1, \ldots, n.$$

where $x_i \in \mathbb{R}^2$ are the position vectors of the bodies, $r_{ij} = |x_i - x_j|$ are their mutual distances, and m_i are their masses. Here the unit of time is taken in order that the Newtonian gravitational constant be equal to one.

The *configuration* of the system formed by the n bodies is denoted by the vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^{2n}$. The differential equations of motion are well–defined when $r_{ij} \neq 0$ for $i \neq j$, i.e. when there is no collisions between the masses.

The dimension of the smallest affine subspace of $\mathbb{R}^2$ which contains all of the points x_i is called the *dimension* $\delta(x)$ *of the configuration* x. Then the configurations with $\delta(x) = 1, 2$ are called *collinear* and *planar*, respectively.

We define the *total mass* and the *center of mass* of the n bodies as

$$M = m_1 + \ldots + m_n, \qquad c = \frac{1}{M}\left(m_1 x_1 + \cdots + m_n x_n\right),$$

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