Commun Nonlinear Sci Numer Simulat 78 (2019) 104872

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Short communication

Equilic quadrilateral central configurations

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ARTICLE INFO

Article history:
Received 6 March 2019
Revised 3 June 2019
Accepted 6 June 2019
Available online 12 June 2019

MSC:
Primary 70f07
Secondary 70f15

Keywords:
Convex central configuration
Four–body problem
Equilic quadrilateral

ABSTRACT

An equilic quadrilateral is a quadrilateral with one pair of opposite sides having the same length, which has angles of inclination whose sum is $2\pi/3$. We characterize the central configurations of the 4–body problem whose four positive masses are at the vertices of equilic quadrilaterals.

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1. Introduction

In the 4-body problem a central configuration occurs when the position vector of each particle with respect to the center of mass is a common scalar multiple of its acceleration vector, and the scalar difference is the same for all particles see for instance [1] and [2].

Several aspects of the 4-body problem motivate the study of central configurations; for instance, they allow to obtain the so called *homographic solutions* of the 4–body problem, which are the unique explicit solutions in terms of the time known until now for that problem, for such solutions the ratios of the mutual distances between the bodies remain constant. Also, central configurations are of utmost importance when studying bifurcations of the surfaces of constant energy and angular momentum, for more details see the papers by Meyer [3] and Smale [4].

The equations of motion of the planar 4–body problem are given by

$$\ddot{x}_i = \sum_{j=1,\ j\neq i}^{4} \frac{m_j(x_j - x_i)}{r_{ij}^3}, \quad \text{for} \quad i = 1, 2, 3, 4$$

where $x_i \in \mathbb{R}^2$ are the position vectors of the bodies, $r_{ij} = |x_i - x_j|$ are their mutual distances, and m_i are their masses. Here the unit of time is taken in order that the Newtonian gravitational constant is set equal to one.

The *configuration* of the system formed by the four bodies is given by the vector $x = (x_1, x_2, x_3, x_4) \in \mathbb{R}^8$. The differential equations of motion are well–defined as long as there are no collisions between the bodies, namely, when $r_{ij} \neq 0$ for $i \neq j$.

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https://doi.org/10.1016/j.cnsns.2019.104872
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