Arch Comput Methods Eng
DOI 10.1007/s11831-010-9048-z

ORIGINAL PAPER

On the Convergence of a Greedy Rank-One Update Algorithm for a Class of Linear Systems

A. Ammar · F. Chinesta · A. Falcó

Received: 15 March 2010 / Accepted: 15 March 2010

Abstract In this paper we study the convergence of the well-known Greedy Rank-One Update Algorithm. It is used to construct the rank-one series solution for full-rank linear systems. The existence of the rank one approximations is also not new, but surprisingly the focus there has been more on the applications side more that in the convergence analysis. Our main contribution is to prove the convergence of the algorithm and also we study the required rank one approximation in each step. We also give some numerical examples and describe its relationship with the Finite Element Method for High-Dimensional Partial Differential Equations based on the tensorial product of one-dimensional bases. We illustrate this situation taking as a model problem the multi-dimensional Poisson equation with homogeneous Dirichlet boundary condition.

This work has been partially supported by the SEJ2006-05051 grant of the Ministerio de Educación y Ciencia and PRCEU-UCH07/08 grant of the Universidad CEU Cardenal Herrera.

A. Ammar (✉)
Arts et Métiers Paris Tech, 2 boulevard du Ronceray, BP 93525, 49035 Angers cedex 01, France
e-mail: Amine.AMMAR@ensam.eu

F. Chinesta
Institut de Recherche en Génie Civil et Mécanique (GeM), Ecole Centrale de Nantes, 1 rue de la Noë, BP 92101, 44 321 Nantes Cédex 3, France
e-mail: Francisco.Chinesta@ec-nantes.fr

A. Falcó
Departamento de Ciencias, Físicas, Matemáticas y de la Computación, Universidad CEU Cardenal Herrera, San Bartolomé 55, 46115 Alfara del Patriarca (Valencia), Spain
e-mail: afalco@uch.ceu.es

1 Introduction

In [1, 2], some of the authors of the present paper propose the use of a separated representation, which allows to define a tensor product approximation basis as well as to decouple the numerical integration of a high dimensional model in each dimension. The milestone of this methodology is the use of shape functions given by a tensorial based construction. This fact has advantages as the manipulation of only one dimensional polynomials and its derivatives, that provides a better computational performance and simplified implementation and use one-dimensional integration rules. Moreover, it makes possible the solution of models defined in spaces of more than hundred dimensions in some specific applications. This problem is closely related with the decomposition of a tensor as a sum of rank-one tensors, that it can be considered as a higher order extension of the matrix Singular Value Decomposition.

The purpose of this work is to formalize and analyze the above strategy in the framework of methods for solving linear systems by means tensor decompositions. As we will show the approximation given in [1, 2], is closely related with the best low-rank approximation problem for high order tensors (see [4]). Unfortunately, in [4] it has been proved that tensors of order 3 or higher can fail to have best rank-r approximation for $r \geq 2$. Our strategy, with the perspective of [4] in mind, is to use the fact that tensors of order 3 or higher have best rank-1 approximation.

In this context, we propose the use of a Greedy Rank-One Update Algorithm to construct, for a full rank linear system, a rank-r approximate solution. This approach is based in the so-called by the signal processing community as the *Matching Pursuit Algorithm* of Mallat and Zhang [12], also known as *Projection Pursuit* by the statistics community (see Friedman and Stuezle [7] and Huber [8]) or as a *Pure*

Published online: 09 October 2010