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## Limit cycles near hyperbolas in quadratic systems

Joan C. Artés<sup>a,1</sup>, Freddy Dumortier<sup>b,\*,2</sup>, Jaume Llibre<sup>a,1</sup>

<sup>a</sup> Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain <sup>b</sup> Mathematics Department, Universiteit Hasselt, Campus Diepenbeek, Agoralaan-Gebouw D, 3590 Diepenbeek, Belgium

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## Abstract

In this paper we introduce the notion of *infinity strip* and *strip of hyperbolas* as organizing centers of limit cycles in polynomial differential systems on the plane. We study a strip of hyperbolas occurring in some quadratic systems. We deal with the cyclicity of the degenerate graphics  $DI_{2a}$  from the programme, set up in [F. Dumortier, R. Roussarie, C. Rousseau, Hilbert's 16th problem for quadratic vector fields, J. Differential Equations 110 (1994) 86–133], to solve the finiteness part of Hilbert's 16th problem for quadratic systems. Techniques from geometric singular perturbation theory are combined with the use of the Bautin ideal. We also rely on the theory of Darboux integrability.

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## 1. Introduction and statement of the results

In studying Hilbert's 16th problem one often considers perturbations from well-known situations. Many papers deal with perturbations from period annuli, a period annulus being a connected set of closed orbits. Especially the period annuli consisting of ellipses have been the

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<sup>\*</sup> Corresponding author.

*E-mail addresses:* artes@mat.uab.cat (J.C. Artés), freddy.dumortier@uhasselt.be (F. Dumortier), jllibre@mat.uab.cat (J. Llibre).

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