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Uniform isochronous cubic and quartic centers: Revisited

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ABSTRACT

In this paper we completed the classification of the phase portraits in the Poincaré disc of uniform isochronous cubic and quartic centers previously studied by several authors. There are three and fourteen different topological phase portraits for the uniform isochronous cubic and quartic centers respectively.

1. Introduction and statement of the main results

The interest in the isochronous centers started in the XVII century with the works of C. Huygens, see [1]. The isochronicity phenomena appear in many physical problems, see for instance [2].

We say that $p \in \mathbb{R}^2$ is a *center* if it is a singular point of a planar differential system such that there is a neighborhood U of p where all the orbits of $U \backslash \{p\}$ are periodic. For every $q \in U \backslash \{p\}$ let $T(q)$ denote the period of the periodic orbit through q. When $T(q)$ is constant for all $q \in U \backslash \{p\}$ we say that p is an *isochronous center*. The fact that p is isochronous does not imply that the angular velocity of the vector $\overrightarrow{pq}$ is the same for all periodic orbits in $U \backslash \{p\}$. When such velocity is constant we say that p is a *uniform isochronous center* or a *rigid center*.

The uniform isochronous planar centers are characterized in the next result.

Proposition 1. *Assume that a planar polynomial differential system of degree n has a center at the origin of coordinates. Then this center is uniform isochronous if and only if by doing a linear change of variables and a scaling of time it can be written as*

$$\dot{x} = -y + x f(x, y), \qquad \dot{y} = x + y f(x, y), \tag{1}$$

with $f(x, y)$ a polynomial in x and y of degree $n - 1$, $f(0, 0) = 0$.

Proposition 1 is well-known, a proof of it can be found in [3].

The next result due to Collins [4] in 1997, also obtained by Devlin, Lloyd and Pearson [5] in 1998, and by Gasull, Prohens and Torregrosa [6] in 2005 characterizes the uniform isochronous centers of cubic polynomial systems.

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