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Quadratic Hamiltonian Vector Fields

JOAN C. ARTÉS AND JAUME LLIBRE

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain

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Let H be a cubic polynomial in two variables over $\mathbb{R}$. Then H defines a quadratic Hamiltonian vector field $(\partial H/\partial y, -\partial H/\partial x)$. The purpose of this paper is to prove that there are exactly 28 non-equivalent topologic phase portraits of quadratic Hamiltonian vector fields.

1. INTRODUCTION

In this paper we study the global phase portraits of *quadratic Hamiltonian vector fields*. That is, the phase portraits of the vector fields $(\partial H/\partial y, -\partial H/\partial x)$ where H is a polynomial of degree 3 in the variables x and y over $\mathbb{R}$.

The characterization of quadratic Hamiltonian vector fields with a center is interesting as a previous step for studying their perturbation and the creation of limit cycles; see, for instance, [B, CJ1, CJ2].

THEOREM 1.1. *Let X be a quadratic Hamiltonian vector field. Then the phase portrait of X is topologically equivalent to one of the* 28 *configurations given in Fig.* 1.1. *Moreover, each of the configurations of Fig.* 1.1 *is realizable by a quadratic Hamiltonian vector field.*

The rest of the paper is dedicated to proving Theorem 1.1. In Section 2 we summarize some results on the (finite) critical points of a quadratic Hamiltonian vector field. All the phase portraits are presented on the closed northern hemisphere of the Poincaré sphere for the compactified quadratic Hamiltonian vector field. For more details and notation on the Poincaré compactification, see [Go, S, ALGM].

Quadratic Hamiltonian vector fields with a center are studied in Section 3 by using Vulpe's classification on quadratic vector fields with a center; see [V]. In Section 4 we reduce the study of quadratic Hamiltonian vector fields to 32 normal forms depending at most on 4 parameters. Finally, Section 5 is dedicated to analysis of the phase portraits of these normal forms.

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