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# ON THE NUMBER OF SLOPES OF INVARIANT STRAIGHT LINES FOR POLYNOMIAL DIFFERENTIAL SYSTEMS*

*Joan C·Artes Jaume Llibre*
(Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, Barcelona, 08193, Spain)

**Abstract** If $P$ and $Q$ are two real polynomials in the real variables $x$ and $y$ such that the degree of $P^2+Q^2$ is $2n$, then we say that the polynomial differential system $x'=P(x,y), y'=Q(x,y)$ has degree $n$. In the set of all polynomial differential systems of degree $n>1$ having finitely many invariant straight lines, let $\alpha(n)$ be the maximum number of invariant straight lines that they have, and let $\beta(n)$ be the maximum number of slopes that these invariant straight lines have. Dai Guoren conjectured that $\beta(n)=2n+(1+(-1)^n)/2$ for $n>2$. In this paper we prove that the conjecture is true for $n=3,4,5$, and that it is not true for $n=6,7,\cdots,21$. Moreover, we prove that $\beta(n)=\alpha(n-1)+1$ and then we refer to [AGL] where $\alpha(n)$ is studied.

**Key words** Polynomial differential system, invariant line, Poincaré sphere.
**AMS(1991) Subject classifications** $O175.12$.

## 1 Introduction and Statement of the Main Results

Let $P$ and $Q$ be two real polynomials in the real variables $x$ and $y$. We say that the polynomial differential system

$$x' = P(x,y), \quad y' = Q(x,y), \tag{1}$$

has degree $n$ if the degree of the polynomial $P^2+Q^2$ is $2n$.

Studies of polynomial differential systems were carried out by Poincaré in [P1],[P2] and [P3]. The algebraic feature of polynomial differential systems renders natural certain questions and problems of an algebraic or an algebra-geometric nature as the following two. Recognize when system (1) has invariant algebraic curves, or is algebraically integrable? See the interesting survey of Schlomiuk [Sc] on these questions. This paper deals with the first question.

The straight line $ax+by+c=0$ is invariant for the flow of system (1), and we call it an

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