Website: http://aimSciences.org

DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS Volume 17, Number 2, February 2007

pp. 259–270

NONEXISTENCE OF LIMIT CYCLES FOR A CLASS OF STRUCTURALLY STABLE QUADRATIC VECTOR FIELDS

J.C. Artés, J. Llibre

Departament de Matemàtiques Universitat Autònoma de Barcelona 08193 Bellaterra, Barcelona, Spain

J.C. Medrado

Instituto de Matemática e Estatística Universidade Federal de Goiás 74001–970 Goiânia, Goiás, Brazil

ABSTRACT. The statistical analysis of the structurally stable quadratic vector fields made in [4] shows that the phase portrait 7.1 (see Figure 1) appears without limit cycles, when the other three phase portraits in the same family with low probability sometimes appear with limit cycles. Here we prove that quadratic vector fields having the phase portrait 7.1 have no limit cycles.

1. Introduction and statement of the main result. A vector field $X : \mathbb{R}^2 \to \mathbb{R}^2$ of the form X = (P, Q) where

$$P = \sum_{0 \le i+j \le 2} a_{ij} x^i y^j \quad \text{and} \quad Q = \sum_{0 \le i+j \le 2} b_{ij} x^i y^j,$$

is called a quadratic (planar polynomial) vector field if

$$\sum_{i+j=2} (|a_{ij}| + |b_{ij}|) \neq 0.$$

Quadratic vector fields have been investigated intensively, and nearly one thousand papers have been published about these vector fields (see, for instance, [14, 19, 20]). But, in general, it is an open problem to know which are the quadratic vector fields having limit cycles.

In [3] the authors characterize the compactified phase portraits of the topologically structurally stable quadratic vector fields without limit cycles. Showing that there are exactly 44 of such phase portraits. Moreover, for the structurally stable quadratic vector fields having limit cycles, they prove that identifying the region bounded by the outermost limit cycle to a single point obtains the phase portrait of a structurally stable quadratic vector field without limit cycles.

²⁰⁰⁰ Mathematics Subject Classification. 58F14, 58F21, 58F30.

Key words and phrases. limit cycles, quadratic vector fields.

^{*} The first two author are partially supported by a DGICYT grant number MTM2005-06098-C02-01 and by a CICYT grant number 2005SGR 00550. The third author is partially supported by a PADCT and CNPq. All authors are also supported by the joint project CAPES-MECD grant 071/04 and HBP2003-0017, respectively.