PIECEWISE LINEAR DIFFERENTIAL SYSTEMS WITH TWO REAL SADDLES

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Abstract. In this paper we study piecewise linear differential systems formed by two regions separated by a straight line so that each system has a real saddle point in its region of definition. If both saddles are conveniently situated, they produce a transition flow from a segment of the splitting line to another segment of the same line, and this produces a generalized singular point on the line. This point is a focus or a center and there can be found limit cycles around it. We are going to show that the maximum number of limit cycles that can bifurcate from this focus is two. One of them appears through a Hopf bifurcation and the second when the focus becomes a node by means of the sliding.

1. Introduction

One of the main problems in the qualitative theory of real planar differential systems is the determination of limit cycles. Limit cycles of planar differential systems were defined by Poincaré [17]. At the end of the 1920s van der Pol [18], Liénard [15] and Andronov [1] proved that a closed orbit of a self–sustained oscillation occurring in a vacuum tube circuit was a limit cycle as considered by Poincaré. After these works, the non-existence, existence, uniqueness and other properties of limit cycles were studied extensively by mathematicians and physicists, and more recently also by chemists, biologists, economists, etc. (see for instance the books [5, 19]). In this paper we are interested in studying the limit cycles of a class of non–smooth differential systems.

A large number of problems from mechanics, electrical engineering and the theory of automatic control are described by non–smooth differential systems, see [2]. The basic methods of qualitative theory for this kind of differential systems were established or developed in the book [9] and in a large number of papers, see for instance [3, 4, 7, 11, 10, 12, 13]. Also the problem of Hopf bifurcation in some of these problems have been studied in [6, 14, 16, 20]. Besides planar linear global differential systems can have at most 10 (7 non degenerated) different phase portraits (see for instance [8]) and no limit cycles, the number of different phase portraits, and the maximum number of limit cycles of piecewise linear systems (even with as few as two regions) is still unknown. The fact that we can situate on each region a linear system with either a real singular point, or a virtual one, plus the fact that another singular point may appear on the splitting line by the combination of the two flows, and the possible existence of limit cycles, increases a lot the number of possible phase portraits of two piecewise linear differential systems.

Consider a planar differential system of the form

$$\dot{x} = f(x, y), \qquad \dot{y} = g(x, y), \qquad x \neq 0 \tag{1}$$

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